Research on the Construction and Application of Higher Education Health Evaluation Index System Based on the AHP-FCE Model

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Abstract: This study aims to construct and apply a comprehensive evaluation index system for assessing the health of higher education using the Analytic Hierarchy Process (AHP) and Fuzzy Comprehensive Evaluation (FCE) models. By integrating multiple indicators into a structured framework, the research categorizes factors into two main fields: Civic Consciousness and National Education Power. The AHP method is employed to determine the weights of these indicators subjectively, while the FCE model transforms qualitative evaluations into quantitative results, providing a systematic and robust approach to assess the health of higher education systems. The study applies this model to six countries, analyzing their higher education health through normalized data and scoring methods, including TOPSIS for optimization. The results reveal significant variations among the selected countries, highlighting the effectiveness of the AHP-FCE model in capturing the complexities of higher education health. This research offers valuable insights for policymakers and educators seeking to improve higher education systems by providing a scientific and comprehensive evaluation tool.

1. Introduction

In order to keep the sustainable development of economy and maintain the competitive-ness in the global marketplace, a nation usually pays more attention to the health of higher education. When the economy has developed to a certain extent, many countries will turn the economic development which is based on light industrial production into the knowledge-skills based economic development. In the field of medicine, there is a clear and quantitative definition for people's health in all aspects, because health is a very basic element for the sustainable development of every individual. Similarly, for the sustainable development of every country, comprehensive and highly educated talents are indispensable. At the same time, when it comes to the education level, should we build a model to complete the evaluation of the health of any nation's system of higher education? The answer is absolutely yes [1].

In the past, scholars tend to analyze and compare the outstanding contributions of colleges and universities in various countries in order to evaluate the corresponding educational achievements. However, we know that the evaluation factors of whether a nation education is healthy are not limited to this. The health of higher education is also affected by the social environment, political factors, economic factors, diplomatic factors and so on. Therefore, the model we need to build needs to be considered from many angles [2, 3].

This study aims to construct and apply a comprehensive evaluation index system for higher education health using the AHP-FCE model. By categorizing indicators into key fields and employing a multi-step evaluation process, this research seeks to provide a systematic and robust framework for assessing the health of higher education systems across different countries. Through the application of this model to a diverse set of nations [4], we aim to offer valuable insights into the strengths and weaknesses of their higher education systems and contribute to the development of evidence-based policies for improving educational quality and national competitiveness.

2. Selection and Classification of Indicators

In assessment of the health of a nation's higher education, we refer to multiple factors. We classify the factors into two main fields: Civic Consciousness and National Education Power. Factors in different fields create influences in different ways. We introduce the quantification of the influence of various factors on the two fields as shown in Figure 1.

Figure 1 Health of higher education index.

2.1. Civic Consciousness

We use three indicators in this field. The three factors have an impact on the civic consciousness about higher education, and they are listed as follows.

Higher education enrollment rate measures the extent of citizens who can get chance to enter universities and colleges. This index objectively and concretely quantifies the importance and popularity of higher education in a country, reflects the education level of its citizens, and has a far-reaching impact on the health and sustainable development of national education. We according to UNESCO Institute for Statistics.

Ratio of male to female in school / ratio of male to female in school age measures whether there is gender discrimination in education in a country, and reflects the fairness of education. In view of the fact that the ratio of men to women in different countries is inconsistent, we do not directly use the ratio of men to women in higher education as the evaluation factor. Instead, we looked up the gender ratio of the average years in school and the ratio of men to women in school age of the country respectively, and compared them to get the evaluation factor. We refer to the data set gender ratio of mean years in school on the World Bank.

Graduates Employability refers to the employment rate and salary of college students after graduation. It can greatly reflect the scarcity of college students in a country's talent market, and also reflects the pressure, social situation and treatment of college students after graduation. It reflects the health level of national education.

2.2. National Education Power

There are five indicators involves multiple sets. These widely used indicators allow us to capture key aspects of national education power.

QS World University Ranking Number is an annual publication of university rankings by Quacquarelli Symonds (QS). It's evaluated according to 40% of acdemic peer review, 20% of Faculty/Student ratio, 20% of Citations per faculty, 10% of emloyer reputation, 5% of International student ratio, and 5% of international staff ratio [5].

The number of Nobel Prize winners. This not only reflects the scientific research level and ability of a country's top talents, but also reflects the overall education and scientific research quality of a country. We acquire the data from 1900 to 2020 on Wikipedia.

Per capita Patent applications states that the creative power and innovation ability the college students have. It reflects the cultivation of talents' innovative ability in nation's education.

Proportion of scientific research input in GDP quantifies the nation's values on science and technology development, which is the basic of national hard power. We refer to the data set research and development expenditure on the World Bank.

Proportion of education investment in GDP quantifies the nation's values on higher education;

It can also objectively reflect the allocation of higher education teachers, the construction of school infrastructure, teachers' salary and other aspects. We refer to the data set Expenditure per student on the World Bank.

We assign 3 years time span for per capita patent applications, 5 years time span for proportion of education investment and proportion of scientific research input in GDP. Due to the frequent occurrence of modern world economic events and policies will not be easily changed within 5 years, its impact will not disappear in an instant. We assume that the latest five-year data predict the investments.

2.2.1. Data Weighting Methods for Individual Indicators

For the two indicators of the first level, namely citizen level and national education power, we find a lot of information, and combined with the background of national higher education system, we get the weight of the first level indicators subjectively. The same method is used to give weight to the three secondary indicators included in the citizen level. As the national education strength includes five secondary indicators, it is often not considerate to consider multiple indicators at one time. So, we adopt the following solution: compare the two indicators, and finally calculate the weight according to the comparison results, that is, use the analytic hierarchy process to determine the weight.

On the basis of analyzing and searching the data, we assume the following judgment matrix as shown in Table 1.

Table 1 Judgment matrix

	EI	**PA**	**RI**	**NP Winners**	**QS**
EI	1	4	2	3	2
PA	1/4	1	1/3	1/2	1/3
RI	1/2	3	1	3	1
NP Winners	1/3	2	1/3	1	1/3
QS	1/2	3	1	3	1

Define the maximum eigenvalue of matrix AA is λ_m. To solve the differential equations in our model, we use in MATLAB find the results. We calculate the consistency index $CI = 0.024368$. For the secondary index included in the national education strength, $n = 5$, so we calculate the consistency ratio $CR = 0.021757$. For $CR < 0.1$, we consider the consistency of the matrix acceptable.

2.2.2. Calculate Weight

In order to ensure the robustness of the results, this paper uses three methods to calculate the weight respectively, and then calculates the scores of each scheme according to the weight matrix, and carries out sorting and comprehensive analysis, so as to avoid the deviation caused by using a single method and make the conclusion more comprehensive and effective.

1) Arithmetic average method

For the first step, we normalize the judgment matrix according to the column, namely to divide each element by the sum of its column). For the second step, we add the normalized columns , namely to sum by row. And the third step is to divide each element of the added vector by n to get the weight vector. The corresponding weight vector formula is as follows:

$$\omega_i = \frac{1}{n}\sum_{j=1}^{n}\frac{a_{ij}}{\sum_{k=1}^{n}a_{kj}}(i = 1,2,\cdots,n) \tag{1}$$

2) Geometric average method

For the first step,we multiply the elements of a by rows to get a new column vector

And then turn each component of the new vector to the n^{th} power. And for the last step is to normalize the column vector to get the weight vector. The corresponding weight vector formula is as follows:

$$\omega_i = \frac{(\prod_{j=1}^{n} a_{ij})^{\frac{1}{n}}}{\sum_{k=1}^{n}(\prod_{j=1}^{n} a_{ij})^{\frac{1}{n}}} \quad (2)$$

3) Eigenvalue method for weight calculation

One eigenvalue of the consistent matrix is n, and the other eigenvalues are 0. In addition, we can easily get that when the eigenvalue is n, the corresponding eigenvector is exactly equals to:

$$k\left[\frac{1}{a_{11}}, \frac{1}{a_{12}}, \cdots, \frac{1}{a_{11}}\right]^T (k \neq 0) \quad (3)$$

which is just the first column of the consistent matrix. The first step is to find the largest eigenvalue of matrix A and its corresponding eigen vector. The second step is to normalize the feature vector, and finally get our weight.

3. Two Hybrid Evaluation Process

To better study the interactions between these several indicators, we combine AHP and fuzzy comprehensive evaluation method to establish a fuzzy comprehensive evaluation (FCE) model based on AHP. Using FCE-AHP model, we can find the health index of higher education system in any nation.

Our model adopts a three-layer structure, the first layer is the target layer, it's purpose is to evaluate the health of a country's higher education system according to the requirements of the topic. Therefore, we need some evaluation indexes as the basis of judgment. After determining the evaluation indexes, we make a simple correlation analysis on these indexes, and then take some indexes with large correlation as a large category to establish a two-level fuzzy comprehensive evaluation model. The construction of any index system must follow certain principles in order to achieve a reasonable, powerful and well-documented index system. This paper fol-lows the following three basic principles: comprehensiveness, representativeness and operability.

3.1. Literature Review

Analytic hierarchy process (AHP): analytic hierarchy process (AHP) is a structured technology for organizing and analyzing complex decision-making based on Mathematics and psychology. It was developed by Thomas L. Saaty in the 1970s, and has been widely studied and improved since then, with special application. Group decision making is widely used in government, business, industry, health care and education all over the world. AHP helps decision-makers to find a goal that best suits them and their understanding of the problem, rather than prescribing a "right" decision. It provides a comprehensive and reasonable framework for constructing decision-making problems, representing and quantifying the elements of decision-making problems, linking these elements with the overall objectives, and evaluating alter-native solutions [6].

Fuzzy comprehensive evaluation (FCE): fuzzy comprehensive evaluation is a comprehensive evaluation method based on fuzzy mathematics. The comprehensive evaluation method based on the theory of fuzzy mathematics transforms the membership degree of qualitative evaluation into quantitative evaluation, and uses fuzzy mathematics to make a comprehensive evaluation of things or objects affected by multiple factors. It has the characteristics of clear and systematic results, can solve the problems of fuzziness and difficult to quantify, and is suit-able for solving the problems of uncertainty. The evaluation results are expressed by fuzzy sets, and the qualitative description and quantitative analysis are closely combined [7].

TOPSIS method: TOPSIS method is a kind of sequential optimization technology with ideal target similarity, which is a very effective method in multi-objective decision analysis. It finds out the best target and the worst target (represented by ideal solution and anti-ideal solution respectively) among multiple targets through normalized data matrix, calculates the distance between each evaluation target and ideal solution and anti-ideal solution respectively, and obtains the closeness degree of each target and ideal solution, which is sorted according to the closeness degree of ideal solution, so as to be the basis for evaluating the quality of targets. When the value of closeness is between 0 and 1, the closer the value is to 1, the closer the corresponding evaluation target is to the optimal level; conversely, the closer the value is to 0, the closer the evaluation target is to the worst level. This method has been successfully applied in many fields, such as land use planning, material selection and evaluation, project investment, medical and health care, etc., which significantly improves the scientificity, accuracy and operability of multi-objective decision analysis.

3.2. Detailed Use of Models

3.2.1. Establishment of FCE Evaluation

1) Establishment of Evaluation Object Factor Set

The evaluation object factor set, also known as the index set, is represented by U in this issue. U consists of two main elements: Civic Consciousness and National Education Power. These two elements are further broken down into more specific evaluation indicators, including enrollment rate, employment rate, the ratio of S to SA, EI, PA, National Prize Winners, and QS. Due to the higher education evaluation system encompassing both the citizen level and the national education power level, these indicators can be further divided into two levels, namely the citizen level. At the citizen level, the evaluation indicators include enrollment rate, employment rate, the ratio of S to SA, EI, PA, National Prize Winners, and QS.

$$U \stackrel{\text{def}}{=} \{u_{11}, u_{12}, u_{21}, u_{22}, u_{23}, u_{24}, u_{25}, u_{26}\} \quad (4)$$

2) Comment Set

The determination of the evaluation set makes the fuzzy comprehensive evaluation to the fuzzy evaluation vector, and the information of the membership degree of each evaluation standard corresponding to the evaluated thing is expressed through the fuzzy vector, which reflects the fuzzy characteristics of the evaluation. In this paper, we define common set $V = \{v_1, v_2, v_3\}$, which corresponding to different levels.

3) Evaluation Standard of Fuzzy Relation

We have searched mass of data and combined with the background of higher education sys-tem in various countries, we have obtained the evaluation criteria for various factors as shown in Table 2.

Table 2 Fuzzy evaluation score.

	v_1	v_2	v_3
u_{11}	0.5(small)	0.65(intermediate)	0.8(big)
u_{12}	5(big)	3.5(intermediate)	2(small)
u_{21}	0.925(small)	095(intermediate)	0.975(big)
u_{22}	0.04(small)	0.05(intermediate)	0.06(big)
u_{23}	100(small)	200(intermediate)	300(big)
u_{24}	0.02(small)	0.03(intermediate)	0.04(big)
u_{25}	50(small)	100(intermediate)	200(big)
u_{26}	2(small)	5(intermediate)	10(big)

4) Membership Function

We choose the trapezoidal distribution as the membership function model because it offers a versatile way to represent uncertain or imprecise data, which is common in many real-world applications. The function diagrams and their corresponding mathematical representations are shown in Figure 2. This figure provides a clear visual and analytical explanation of how the trapezoidal membership function operates across different scenarios, illustrating the transition of membership values from 0 to 1 and back to 0, which is crucial for fuzzy logic systems where precise boundaries are often not available or practical.

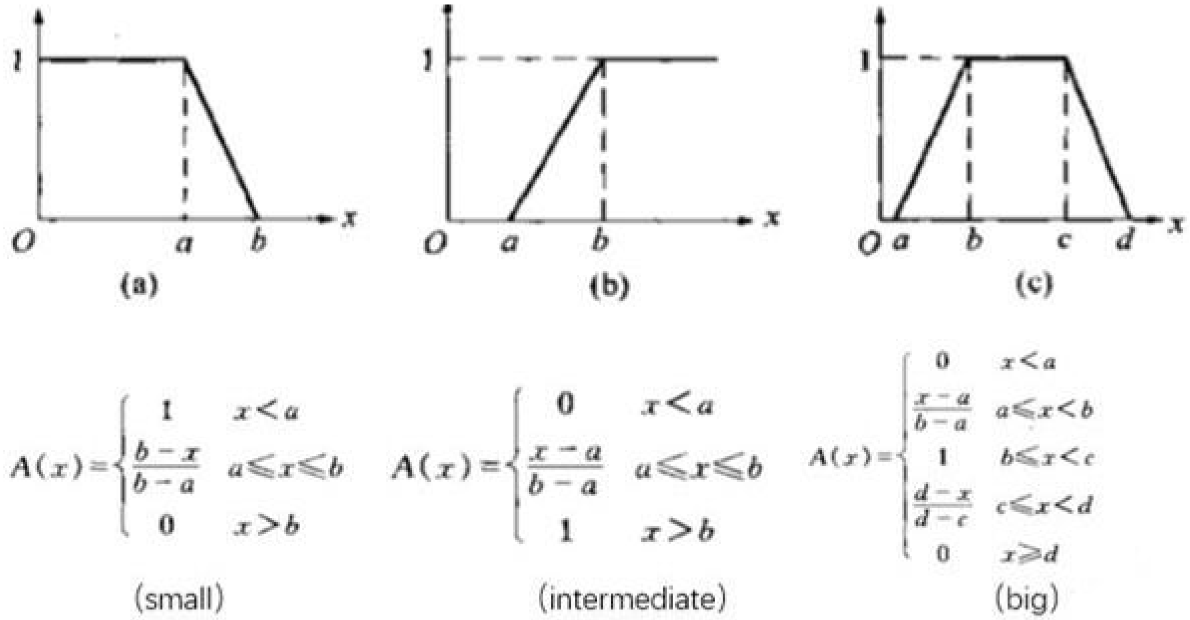

Figure 2 Trapzoidal membership function model and its application examples.

For each evaluation factor, different comments have a specific membership function. The evaluation indexes a, B and C in the function model are given in the "criteria for determining fuzzy evaluation relationship" above. Therefore, we determine 24 membership functions with different comments for each factor.

5) Establishing fuzzy relation matrix R

By establishing the membership function R, we can put the specific data of the evaluation system into each evaluation object, that is, different countries. The establishment of R is as follows.

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nm} \end{bmatrix} \tag{5}$$

Where $r_{ij}(i = 1,2,\dots,n, j = 1,2,\dots,m)$ is the membership degree of the $i-th$ index in $object\ factor\ U$ corresponding to the $j-th$ evaluation standard in common set V.

3.2.2. Establishment of AHP Evaluation

When we try to get the weight of five aspects as the weight of secondary indicators, for subjective judgment is easy to make mistakes, so in this part we use AHP as an important method to determine the weight of indicators in the evaluation system. The model architecture is shown in Figure 3.

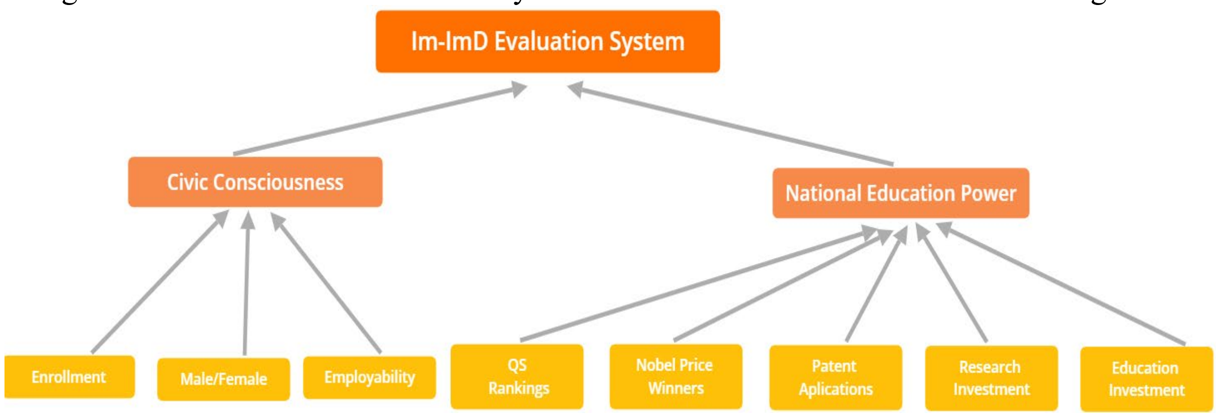

Figure 3 AHP methods.

1) Data Weighting Methods for Individual Indicators

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2) Consistency test of judgment matrix

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3) calculation weight

In order to ensure the robustness of the results, this paper uses three methods to calculate the weight respectively, and then calculates the scores of each scheme according to the weight matrix, and carries out sorting and comprehensive analysis, so as to avoid the deviation caused by using a single method and make the conclusion more comprehensive and effective.

4. Results

4.1. Model Application and Evaluation

We use the Im-ImD Model to determine the health of a nation. After mass of data screen-ing, we selected six countries to calculate specific weights. Through FCE method, we can finally get the weight set of those eight factors. The results are listed in Table 3.

Table 3 Weight Set

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
0.16	0.12	0.12	0.221	0.043	0.137	0.062	0.137

After multiplying the weight by the normalized data, we get the score as shown in Table 4.

Table 4 Score from FCE

	General	**Good**	**Excellent**
China	0.5063	0.3263	0.1674
Japan	0.4907	0.4085	0.1035
America	0.1410	0.1590	0.1035
England	0.3784	0.3513	0.2703
France	0.5680	0.3513	0.2703
Korea	0.4390	0.3378	0.2232

Through TOPSIS method, we normalize the data after AHP to get the corresponding results. Then, we define the standardized matrix is Z, for every element in Z,$z_{ij} = x_{ij}/\sqrt{\sum_{i=1}^{n} {x_{ij}}^2}$.Then, we get the unormalized score formula:

$$Score_{un} = \frac{\sqrt{\sum({Z_j}^+ - z_{ij})^2}}{\sqrt{\sum({Z_j}^+ - z_{ij})^2} + \sqrt{\sum({Z_j}^- - z_{ij})^2}} \tag{6}$$

At last, by dividing each term by the terms and adding them, we get the normalized result in Table 5.

Table 5 Normalized Score

	Enrollment	Employ	S/SA	EI	PA	RI	NP Winners	QS
China	0.32175	0.26968	0.43608	0.33602	0.21378	0.30372	0.00739	0.27217
Japan	0.39601	0.40452	0.407	0.33602	0.47444	0.45557	0.06902	0.09072
America	0.53832	0.6742	0.40285	0.49993	0.34317	0.40035	0.92928	0.86186
England	0.30938	0.53936	0.41092	0.41798	0.06001	0.24849	0.32044	0.40825
France	0.41457	0.13484	0.39797	0.42617	0.04688	0.30372	0.17008	0
Korea	0.42694	0	0.39326	0.40978	0.77823	0.62124	0.00246	0.09072

Using the formula to calculate the score, we can get the results as shown in Table 6.

Table 6 Final Score

China	Japan	US	UK	France	Korea
0.1055	0.1501	0.3351	0.1763	0.0658	0.1671

4.2. Result Visualization and Analysis

By visualizing the final data, Figure 4 and Figure 5 reflect the health of higher education in six countries.

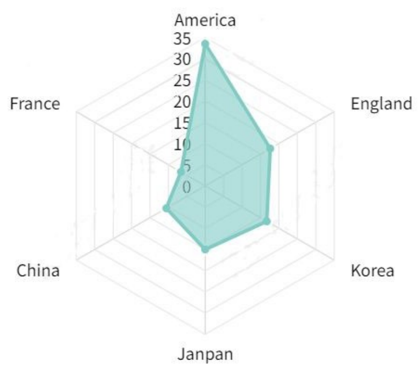

Figure 4 Score Radar Chart.

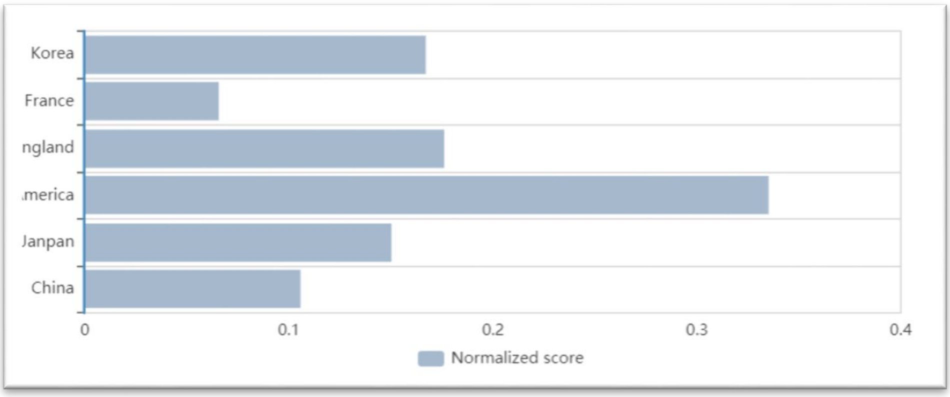

Figure 5 Score Bar Chart.

5. Conclusion

In this study, we have developed and applied a comprehensive evaluation framework for assessing the health of higher education systems using the AHP-FCE model. The integration of multiple indicators into a structured framework has allowed us to provide a holistic and nuanced assessment that captures the multifaceted nature of higher education health. The AHP method has been particularly effective in prioritizing and weighting various indicators based on their relative importance, while the FCE approach has successfully transformed qualitative assessments into quantitative results, accommodating the inherent uncertainties and subjectivity in evaluating complex systems. The application of this model to six countries has demonstrated its ability to identify significant variations in higher education health, highlighting its potential for comparative analysis and policy-making.

However, despite its strengths, the model also has limitations that warrant further exploration. The weight assignment in AHP relies heavily on expert judgment, which introduces a degree of subjectivity. Future work could explore more objective methods for weight determination, such as data-driven approaches or the integration of expert consensus techniques, to enhance the model's reliability. Additionally, the model currently relies on static data sets, which may not fully capture the dynamic nature of higher education systems and external influences. Incorporating dynamic data sources and real-time updates could provide a more accurate and up-to-date evaluation. Furthermore, while the model offers valuable insights for individual countries, its applicability for cross-country comparisons could be improved by standardizing data sources and ensuring consistency in indicator definitions across different contexts.

Overall, the AHP-FCE model has proven to be a robust tool for evaluating higher education health, offering a balanced and comprehensive assessment. Future research should focus on addressing the identified limitations to enhance the model's objectivity, adaptability, and cross-country comparability. By doing so, we can further refine this framework to better support evidence-based policy-making and contribute to the continuous improvement of higher education systems globally.

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